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Heat transfer in the pneumatic transport of massive particles

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Abstract—The steady, fully developed heat transfer to the walls of a vertical pipe from a dilute suspension of relatively massive particles of low Biot number in a turbulent gas is analyzed. In this flow, particle collisions play a significant role. The thermal energies of the particle and gas phases are balanced using two coupled equations. In the particle phase, conduction is calculated from the kinetic theory as a self-diffusive transport flux and, assuming negligible transfer of heat during collisions, homogeneous boundary conditions are prescribed for the temperature. Solutions of the balance laws highlight the mechanisms governing the heat transfer in this regime.

1. INTRODUCTION

BECAUSE accurate predictions of heat transfer are essential for the design of pneumatic transport lines, measurements of effective heat transfer rates through the wall have been reported in gas–solid suspensions at high velocity (e.g. Jepson *et al.* [1], Soo [2], Boothroyd and Haque [3]) and phenomenological models have been proposed to correlate the resulting data with local flow parameters [4]. However, because detailed studies of turbulent suspensions addressed only the dilute flow of small, non-interacting particles (e.g. Elghobashi and Abou-Arab [5], Pourahmadi and Humphrey [6]), physical analyses of the energy transfer were limited to regimes where particle collisions play a minor role in the flow (e.g. Derevich *et al.* [7]).

In a recent paper, Louge *et al.* [8] consider dilute suspensions of massive particles in a gas and show that collisions are essential to predict the transfer of momentum and fluctuating energy in the particle phase. In their analysis, the particles exchange momentum through collisions with the wall and between themselves, while the mean drag exerted by the gas suspends the particles. However, the particles are assumed to be massive enough to be unaffected by the turbulent velocity fluctuations. The turbulence is modeled using a one-equation closure that incorporates contributions from the particle phase.

In the present study, we analyze fully developed heat transfer in the same regime. To this end, separate thermal energy equations are employed for the particle and the gas phase. These are coupled through a source term calculated by averaging a heat transfer correlation for a single particle. In this flow the heat transferred to particles in and between collisions is negligible. Consequently, conduction in the dilute particle phase is treated as a self-diffusive transport flux derived from the kinetic theory and homogeneous boundary conditions are prescribed for the particle phase at the wall. The self-diffusive transport of thermal energy in the particle phase results from the random motion of particles with different temperatures across a surface in the flow. It is analogous to the self-diffusive molecular transport in a dilute hard-sphere gas. The equations are solved numerically for constant wall heat flux or constant wall temperature to predict heat transfer rates at the wall and temperature profiles across the flow. The resulting predictions are compared with the measurements of Jepson *et al.* [1].

We begin with a summary of the hydrodynamic treatment for the fully developed flow of massive particles in a pipe. Then, through an analysis of the thermal energy balance of the two phases, we isolate the parameters that govern the steady, fully developed heat transfer in the same regime. Because the presence of particles tends to reduce temperature variations across the pipe, we assume that all material and fluid properties are constant, so that the earlier hydrodynamic predictions [8] may be employed directly in the heat transfer analysis.

2. HYDRODYNAMIC ANALYSIS

Louge *et al.* [8] consider the dilute, fully developed, steady flow of particles in a vertical pipe. They focus on particles massive enough to be unaffected by the velocity fluctuations in the turbulent gas. For such particles, the hydrodynamic relaxation time of the

NOMENCLATURE

Ar	Archimedes number, $\rho_p \rho g d^3/\mu^2$
c	gas heat capacity per unit mass
c_p	particle heat capacity per unit mass
C_d	drag coefficient
C_μ	coefficient, 0.49
C_2	coefficient, C_μ^3
d	particle diameter
D	pipe diameter
D_1	collisional rate of energy dissipation per unit volume
D_2	working of the fluctuating drag per unit volume on $\mathbf{v}'$
e	particle–particle coefficient of restitution
e_w	particle–wall coefficient of restitution
E	isotropic turbulent dissipation rate
g	acceleration of gravity
h	wall heat transfer coefficient
H	rate of gas–particle heat exchange per unit volume
k	gas turbulent kinetic energy per unit mass
k_g	gas molecular conductivity
k_p	particle conductivity
l	turbulent mixing length
m	loading, i.e. ratio of particle to gas mass flow rates
M	total heat capacity rate
n	particle number density
N	particle normal stress
Nu	particle Nusselt number
Nu_D	pipe Nusselt number, hD/k_g
p	gas pressure
Pr	gas Prandtl number
Pr_t	turbulent Prandtl number, $\varepsilon_H/\varepsilon_M = 0.9$
q	flux of particle granular temperature
q_w	wall heat flux
Q	average gas mass flow rate
Q_r	radial gas heat flux
Q_z	axial gas heat flux
Q_r^p	radial particle heat flux
Q_z^p	axial particle heat flux
r	radial coordinate
R	pipe radius
Rc	ratio, τ_e/τ_c
Re	particle Reynolds number, $\lvert\mathbf{S}\rvert \rho d/\mu$
Re_D	pipe Reynolds number
Re_p	mean slip Reynolds number, $\lvert\mathbf{u}-\mathbf{v}\rvert \rho d/\mu$
$\mathbf{s}$	average slip velocity vector, $\mathbf{u}-\mathbf{v}$
$\mathbf{s}'$	fluctuating slip velocity vector
S	particle shear stress
$\mathbf{S}$	instantaneous particle slip velocity vector
S_0	particle shear stress at the wall
T^b	average bulk temperature
T_g	average gas temperature
T_p	average particle temperature
T_w	wall temperature
T_g^b	gas bulk temperature (44a)
T_p^b	particle bulk temperature (44b)
u	mean gas velocity along the vertical axis
u_i'	fluctuating gas velocity component
u^*	shear velocity
v	mean particle velocity along the vertical axis
$\mathbf{v}'$	particle velocity fluctuation vector
v_i'	particle fluctuating velocity component
y^+	dimensionless wall coordinate, $\rho(R-r)u^*/\mu$
z	upward vertical coordinate.

Greek symbols

α	gas molecular heat diffusivity, $k_g/\rho c$
Δ	combination of E and σ
ε	voidage
$\bar{\varepsilon}$	average cross-sectional voidage
ε_H	eddy diffusivity of heat
ε_M	eddy diffusivity of momentum
E	Young's modulus
θ_g	normalized average gas temperature (38a)
θ_p	normalized average particle temperature (38b)
Θ	granular temperature
κ	von Kàrmàn's constant, 0.41
λ	particle mean free path
μ	gas viscosity
μ_f	particle–wall Coulomb friction coefficient
μ_t	turbulent eddy viscosity
ξ	function in equation (30)
ρ	gas density
ρ_p	particle material density
σ	Poisson's ratio
σ_k	turbulent Prandtl number for k
τ	gas shear stress
τ_0	gas shear stress at the wall
τ_c	collision time
τ_e	characteristic time for conduction heat transfer
T	hydròdynamic relaxation time.

particle velocity fluctuations is much greater than a typical roll-over time of the turbulent eddies that is based on their integral length scale l and their root-mean-square (r.m.s.) turbulent velocity u'. Because the turbulent eddy size is of the order of the pipe diameter, such a flow would typically be observed in pipes of relatively small dimensions. In this case, the particles are suspended by the drag force exerted by the mean gas flow, but their velocity fluctuations are the result of collisions with other particles or with the wall.

For the particle phase, these authors adopt the assumption of molecular chaos central to the treatment of collisions in rapid granular flows [9] and assume that interparticle collisions are nearly elastic and frictionless. They also focus on flows where a particle undergoes at least several collisions during its hydrodynamic relaxation time. Consequently, a particle loses only a small fraction of its fluctuation energy in and between collisions and the velocity distribution function for the particles is nearly Maxwellian. Finally, they focus on fully developed, steady flows, which permits them to carry out averages for both the gas and particle properties on long vertical strips. Then, no matter how dilute the flow, the control volumes can always include a large enough number of particles to define the appropriate averages, so the dispersed particles may be regarded as a continuum. For the particles, they assume that the averages on the strips are equivalent to averages based on a velocity distribution function.

With these assumptions, well-established results from the kinetic theory of gases lead to constitutive relations for the pressure and shear stress in the particle phase [10]. The resulting momentum equation for the particles is then modified to incorporate the drag force from the gas and the gravitational force. For a dilute, fully developed, axisymmetric flow, the vertical component of this momentum balance is

$$0 = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rS) + \frac{\rho_p}{\mathrm{T}}(1-\varepsilon)(u-v) - \rho_p(1-\varepsilon)g \quad (1)$$

where u and v are, respectively, the vertical components of the mean gas and particle velocities, r the radial coordinate, g the gravitational acceleration, ε the voidage, ρ_p the material density of the particles, T the hydrodynamic relaxation time of the mean relative (slip) velocity between the phases, and S the particle shear stress on surfaces at constant radius.

The shear stress is given by

$$S = \frac{5\sqrt{\pi}}{96}\rho_p d\sqrt{\Theta}\frac{\mathrm{d}v}{\mathrm{d}r}\frac{1}{1+\lambda/R}. \quad (2)$$

Here, Θ is the 'granular temperature', expressed in terms of the r.m.s. particle velocity fluctuations $\mathbf{v}'$ as $(3/2)\Theta = (1/2)\mathbf{v}'^2$. Note that this 'temperature' is a hydrodynamic measure of the agitation of the grains that bears no relation to the conventional thermal temperature of the particles. Also, the factor λ/R in the denominator is a heuristic correction for the very dilute flow of relatively large particles that is important when the length of the mean free path between collisions $\lambda = d/[6\sqrt{2}(1-\varepsilon)]$ is comparable to the radius R of the pipe.

The hydrodynamic relaxation time is defined in terms of the drag force on one particle by

$$\frac{\rho_p}{\mathrm{T}} = C_d|u-v|\frac{3\rho}{4d} \quad (3)$$

where ρ is the gas density and d is the particle diameter. For the drag coefficient C_d, the empirical expression

$$C_d = \frac{24}{Re_p}(1+0.15Re_p^{0.687}) \quad (4)$$

is valid in the range $0 < Re_p < 800$, where $Re_p = |u-v|\rho d/\mu$ is the Reynolds number based on the average slip velocity between the phases and μ is the viscosity of the gas [11].

In these dilute flows, the particle momentum balance in the radial direction becomes

$$\mathrm{d}N/\mathrm{d}r = 0 \quad (5)$$

where N is the particle pressure, which is related to the particle volume fraction $(1-\varepsilon)$ by an expression analogous to an equation of state in a molecular gas:

$$N = \rho_p(1-\varepsilon)\Theta. \quad (6)$$

In order to close the equations describing the particle phase, a balance of fluctuating energy is written to determine the granular temperature Θ. The energy equation derived by Jenkins and Savage [9] is, for fully developed flow

$$0 = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rq) + S\frac{\mathrm{d}v}{\mathrm{d}r} - D_1 - D_2. \quad (7)$$

Here q is the diffusive flux of particle fluctuation energy and D_1 and D_2 are rates of dissipation of the fluctuating energy per unit volume due, respectively, to the inelasticity of the particles and to their interaction with the gas. The energy flux is given by the kinetic theory as

$$q = \frac{25\sqrt{\pi}}{128}\rho_p d\sqrt{\Theta}\frac{\mathrm{d}\Theta}{\mathrm{d}r}\frac{1}{1+\lambda/R} \quad (8)$$

where the correction for long mean free path is analogous to that in (2). In the dilute limit, for nearly elastic particles

$$D_1 = \frac{24(1-e)\rho_p}{\sqrt{\pi}d}\Theta^{3/2}(1-\varepsilon)^2 \quad (9)$$

where e is the coefficient of restitution for a particle–particle collision [9]. The rate of energy dissipation per unit volume D_2 results from the working of the fluctuating force exerted by the gas through the fluctuating velocity of the particles. It may be written as

$$\begin{aligned} D_2 &= -(\rho_p/\mathrm{T})(1-\varepsilon)\overline{v_i'(u_i'-v_i')} \\ &= -(\rho_p/\mathrm{T})(1-\varepsilon)(\overline{u_i'v_i'}-3\Theta) \end{aligned} \quad (10)$$

where primes indicate fluctuating velocities and the overbar denotes the average. The term $\overline{u_i'v_i'}$ is the correlation between the velocity fluctuations of the gas and those of the particles. For this correlation, the expression of Koch [12] is extended to other than low particle Reynolds numbers using a relaxation time T_1 based on the r.m.s. fluctuating slip velocity between the two phases:

$$\overline{u_i'v_i'} = \frac{4}{\sqrt{\pi}}\frac{d(u-v)^2}{T_1\sqrt{\Theta}}. \quad (11)$$

For the boundary conditions of the particle phase at the wall, we adopt expressions calculated by Jenkins [13] in the limit where the coefficient of friction μ_f is so small that the point of contact of a particle always slips during a collision:

$$S = -\mu_f N \tag{12a}$$

and

$$q = \tfrac{3}{8}N\sqrt{(3\Theta)}[\tfrac{7}{2}(1+e_w)\mu_f^2-(1-e_w)] \tag{12b}$$

where e_w is the coefficient of restitution for a particle colliding with the wall.

In the gas phase the momentum equation for fully developed, axisymmetric flow is

$$0 = \frac{1}{r}\frac{d}{dr}(r\varepsilon\tau) - \frac{\rho_p}{T}(1-\varepsilon)(u-v) - \frac{\partial(\varepsilon p)}{\partial z} \tag{13}$$

where $\tau = (\mu+\mu_t)\,du/dr$ is the gas shear stress on vertical surfaces at constant radius and the eddy viscosity μ_t is assumed to be given by the one-equation closure described, for example, by Reynolds [14]:

$$\mu_t = C_\mu \rho\sqrt{k}l. \tag{14}$$

Here l is the turbulent mixing length, $C_\mu = 0.49$, and $k = \overline{u_i'u_i'}/2$ is the turbulent kinetic energy per unit mass of the gas. Following common practice in pipe flow, the analysis in [8] assumes that near the wall the mixing length is proportional to the distance from the wall and that near the center of the pipe it is constant:

$$l/R = \begin{cases} \kappa(1-r/R), & r/R \geq 0.7 \\ 0.3\kappa, & r/R \leq 0.7 \end{cases} \tag{15}$$

where $\kappa = 0.41$ is von Kármán's constant.

In addition, the balance of turbulent kinetic energy k is obtained from that of Elghobashi and Abou-Arab [5] by ignoring the voidage fluctuations ε', which are zero for the steady velocity distribution function, and by suppressing other terms on the basis of an order-of-magnitude analysis [8]. For fully developed flow, the result is

$$0 = \frac{1}{r}\frac{d}{dr}\left\{r\varepsilon\left(\frac{\mu_t}{\sigma_k}+\mu\right)\frac{dk}{dr}\right\} + \mu_t\varepsilon\left(\frac{du}{dr}\right)^2 - \rho\varepsilon \mathrm{E} - \frac{\rho_p}{T}(1-\varepsilon)(2k-\overline{u_i'v_i'}). \tag{16}$$

The isotropic dissipation rate E is modeled after Reynolds [14] using $\mathrm{E} = C_2 k^{3/2}/l$, where $C_2 = C_\mu^3$. Following common practice we adopt $\sigma_k = 1$.

Boundary conditions for the gas phase are enforced at a dimensionless distance $y^+ = \rho(R-r)u^*/\mu \approx 30$ away from the wall, where $u^* = \sqrt{(\varepsilon\tau_0/\rho)}$ is the shear velocity and τ_0 is the gas shear stress at the wall. In dilute flow, the gas velocity at small values of y^+ is approximately given by the universal 'law of the wall':

$$u/u^* = \begin{cases} 5\ln y^+ - 3, & 5 \leq y^+ \leq 30 \\ (1/\kappa)\ln y^+ + 5.7, & y^+ \geq 30 \end{cases}. \tag{17}$$

The shear velocity is calculated from the global momentum balance:

$$\partial(\varepsilon p)/\partial z = -2\rho u^{*2}/R + 2S_0/R - g\rho_p(1-\bar{\varepsilon}) \tag{18}$$

where S_0 is the particle shear stress evaluated at the wall and $\bar{\varepsilon}$ the average voidage across the pipe. Because the production and dissipation of k are nearly equal near the wall, and because the molecular viscosity is much smaller than the eddy viscosity at $y^+ \approx 30$, there is no flux of turbulent kinetic energy there, so

$$\partial k/\partial r = 0. \tag{19}$$

Finally, radial symmetry implies that the radial derivative of the velocities and the temperatures vanish at the centerline of the pipe.

The resulting non-linear, coupled, two-point boundary value problem of equations (1), (5)–(7), (13), (16) subject to the boundary conditions (17), (19) are solved using the quasi-linearization method of Bellman and Kalaba [15]. In Figs. 1 and 2, the results are compared with velocity and turbulent kinetic energy profiles measured by Tsuji et al. [16] for typical conditions. In these the loading m is defined as the ratio of the particle to gas mass flow rates. Because a treatment that ignores shear stress in the balance of forces for the particle phase (dashed line in Fig. 1) clearly fails to reproduce the observed velocity profile, we conclude that particle collisions play an essential role in these flows.

In addition, the data of Tsuji et al. [16] suggest that the average particle velocity is positive at the wall. Because the gas velocity is zero there, particles with relatively small terminal velocities may acquire an average velocity greater than that of the gas over a

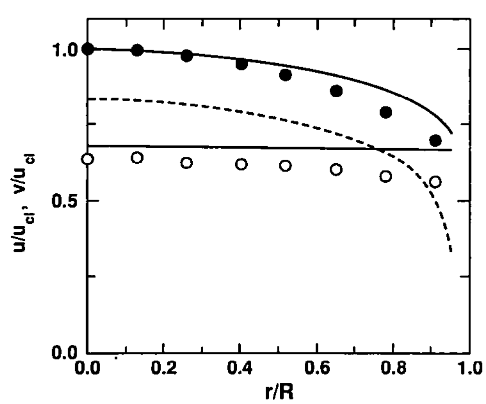

FIG. 1. Calculated profiles of gas and particle velocities normalized by the centerline air velocity $u_{cl} = 9.65$ m s^{-1} for relatively dilute flows of 500 μm polystyrene particles ($\rho_p = 1.02$ g cm^{-3}) at a loading $m = 1.1$. The solid and open circles represent the data of Tsuji et al. [16] for gas and particle velocities, respectively. The dashed lines represent particle velocities predicted by an analysis that would ignore particle shear. In all calculations we adopt a coefficient of restitution $e = 0.9$ for particle–particle collisions, $e_w = 0.7$ for particle–wall collisions, and a coefficient of dynamic friction $\mu_f = 0.2$ between a particle and the wall.

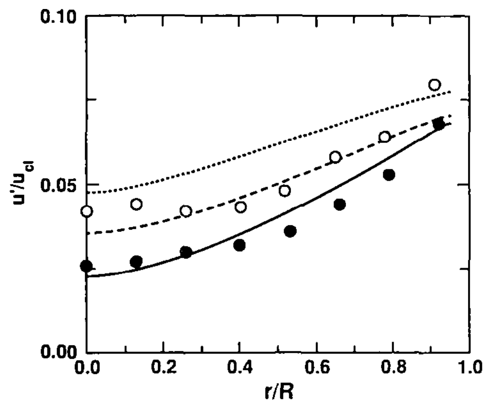

FIG. 2. Calculated profiles of normalized r.m.s. gas velocity fluctuations u'/u_{cl} for relatively dilute flows. The solid circles represent the data of Tsuji *et al.* [16] for 200 μm particles, $u_{cl} = 12.8$ m s^{-1}, $m = 1.3$; the open circles are 500 μm particles, $u_{cl} = 13.3$ m s^{-1}, $m = 1.3$. The solid and dashed lines are the respective predictions of the analysis, which also yields $(1-\bar{\varepsilon}) = 0.16\%$, $\Theta^{1/2}/u_{cl} \approx 1.8\%$ for 200 μm particles, and $(1-\bar{\varepsilon}) = 0.18\%$, $\Theta^{1/2}/u_{cl} \approx 2\%$ for 500 μm particles. The dotted line is the prediction for clear gas.

significant region away from the wall, see for example the 200 μm particles employed by Tsuji *et al.* The momentum transport in the particle phase provides the mechanism that is responsible for this (ref. [8], Fig. 2(a)). The moderate discrepancies between the measured and the predicted velocity profiles near the wall appear to be associated with an over-prediction of the magnitude of the particle phase viscosity. This effect may result from the admittedly crude rare gas correction in equations (2) and (8).

Finally, the hydrodynamic analysis also predicts a reduction in the turbulent velocity fluctuations associated with the presence of particles and that this reduction disappears with increasing particle diameter (Fig. 2). These effects are primarily related to the modifications of the mean gas velocity profile. Because the drag term in equation (13) increases with particle volume fraction, the introduction of particles flattens the gas velocity profile in the interior, so the production of turbulent kinetic energy through the working of the mean gas shear in (16) decreases there. In addition, because the drag term decreases strongly with increasing particle diameter, the reduction of the turbulent fluctuations is less pronounced for large particles. These trends will be particularly important in the thermal analysis that follows.

3. THERMAL ANALYSIS

In this section, we write balance laws for the thermal energy of the gas and particle phases. We assume that particles are massive enough for the flow to lie within the regime studied by Louge *et al.* [8] but have Biot numbers small enough to ignore temperature variations in their interior.

3.1. *Gas phase*

In a dilute turbulent flow, the thermal energy balance for the gas phase has nearly the same form as the corresponding equation for the pure gas. It is obtained by multiplying appropriate terms by the voidage. Because in fully developed flow radial velocities vanish, it becomes

$$\varepsilon\rho c u \frac{\partial T_g}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(rQ_r) + \frac{\partial}{\partial z}(Q_z) + H \qquad (20)$$

where c is the heat capacity per unit mass of the gas, Q_r and Q_z heat fluxes through the gas in the radial and axial direction, respectively, and H the average rate of energy per unit volume supplied by the particle phase to the gas. In this formulation, T_g is the temperature of the gas averaged over time. In keeping with the simple turbulence closure in (14) and (15), we employ an eddy diffusivity ε_H to relate the correlation between the instantaneous fluctuations of temperature and velocity of the gas to the temperature gradient. In this case, heat fluxes take the form:

$$Q_r = (k_g + \rho c \varepsilon_H)\varepsilon \frac{\partial T_g}{\partial r} \qquad (21a)$$

and

$$Q_z = (k_g + \rho c \varepsilon_H)\varepsilon \frac{\partial T_g}{\partial z} \qquad (21b)$$

where k_g is the gas conductivity. Following common practice, we assume that the turbulent Prandtl number $Pr_t \equiv \varepsilon_M/\varepsilon_H$ is 0.9, where $\varepsilon_M \equiv \mu_t/\rho$ is the eddy diffusivity of momentum.

3.2. *Particle phase*

For nearly elastic particles, a simple analysis permits us to ignore the heat exchanged by thermal conduction during a collision between a particle and the wall. Using the elastic collision model of Timoshenko and Goodier [17], we compare estimates of the collision time τ_c with the time τ_e to equilibrate the particle temperature by conduction through the area of contact. By approximating the relative velocity of the particles before collision as the square root of the granular temperature, these estimates are written in terms of average flow parameters

$$\tau_c \sim \frac{\rho_p^{2/5}\Delta^{2/5} d}{\Theta^{1/10}}$$

and

$$\tau_e \sim \frac{\rho_p^{3/5} d^2 c_p}{\Delta^{2/5}\Theta^{2/5} k_p}$$

where c_p and k_p are the particle's heat capacity per unit mass and thermal conductivity, respectively, and $\Delta = 3(1-\sigma^2)/2E$ combines Young's modulus E and Posson's ratio σ for the solid particle. Here, the heat conducted during a collision may be ignored as long as the following ratio is large:

$$Rc \equiv \frac{\tau_c}{\tau_c} \sim \frac{\rho_p^{1/5} d c_p}{\Delta^{4/5}\Theta^{3/10} k_p} \gg 1. \tag{22}$$

Because for the flows under consideration, Rc is of the order 10^6–10^9, the solid thermal conductivity plays no role in the mechanism of heat transfer, and a particle exchanges virtually no heat during its nearly instantaneous collisions with the wall. These observations are consistent with the more detailed theoretical analysis of Sun and Chen [18]. This argument implies that the flux of thermal energy from the wall to the particles is zero. Then, because the granular temperature does not vanish in the neighborhood of the wall, the gradient of the thermal temperature in the particle phase must be zero there.

In a wider context, because experience shows that heat transfer rates are often a weak function of the thermal conductivity of the solid material, the transfer of heat by conduction through particles in direct contact is generally negligible in gas–particle suspensions at high velocities [19].

The establishment of a balance equation for the thermal energy of the particle phase requires the definition of an average particle temperature T_p. Because unlike other particle flow parameters, the thermal energy of the particle phase varies along the pipe, we cannot adopt a definition of T_p based on long vertical strips. Instead we assume that time or ensemble averages of the particle thermal temperature carried out at a point are equivalent to the strip averages employed in the hydrodynamic analysis. In this case the balance equation for the thermal energy of the particle phase is

$$(1-\varepsilon)\rho_p c_p v \frac{\partial T_p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r Q_r^p) + \frac{\partial}{\partial z}(Q_z^p) - H. \tag{23}$$

In such dilute systems, the radial and longitudinal fluxes Q_r^p and Q_z^p arise from the self-diffusive transport of thermal energy carried by the fluctuating particles, and the average transfer of heat between colliding particles is small compared to the self-diffusive flux. In the context of the kinetic theory, this self-diffusive transport of particle thermal energy is analogous to the self-diffusive molecular transport in a dilute hard-sphere gas. We will refer to it as the 'particle self-diffusive conduction'. In general, because particles and gas have different thermal temperatures, they may exchange energy during this transport. The product of the particle collision frequency and a characteristic time for this transfer of heat is of order $[\Theta^{1/2}(1-\varepsilon)(1+\lambda/R)]c_p \rho_p d/k_g$. Because for the flows under consideration this product varies between 4 and 10^2, it is large enough to neglect the energy exchange between collisions and, in these dilute, hardly dissipative flows, the particle fluxes of thermal energy Q_r^p and Q_z^p may be derived from the self-diffusion of a hard-sphere gas [10]

$$Q_r^p = \frac{\sqrt{\pi}}{16}\rho_p c_p d\sqrt{\Theta}\frac{\partial T_p}{\partial r}\frac{1}{1+\lambda/R} \tag{24a}$$

and

$$Q_z^p = \frac{\sqrt{\pi}}{16}\rho_p c_p d\sqrt{\Theta}\frac{\partial T_p}{\partial z}\frac{1}{1+\lambda/R} \tag{24b}$$

where the correction for long mean free path is analogous to that in (2) and (8). Note that, as a consequence of this correction, the fluxes approach zero as $(1-\varepsilon)$ vanishes, so that equation (23) remains valid in this limit.

In more concentrated systems of particles the contribution of the particle self-diffusive conduction to the energy flux is diminished and collisional contributions to the particle energy flux due to conduction through contacts become, potentially, more important. However when the ratio Rc in (22) is much greater than one, the collisional contribution to the thermal energy flux in the particle phase is negligible even in concentrated systems. In this case, the net result of increasing the concentration is to decrease the total energy flux in the particle phase.

3.3. *Source term*

The source term H in equations (20) and (23) averages the contribution from all particles to the thermal energy of the gas. Because the fluctuating velocities of the gas and the particles are much smaller than their average values [8], the flow around each particle may be regarded as nearly steady. Consequently, to evaluate the heat convected away from each particle in dilute flows, we adopt a correlation appropriate for a single sphere in a steady, uniform, infinite gas stream [20]:

$$Nu = 2 + (0.4Re^{1/2} + 0.06Re^{2/3})Pr^{0.4} \tag{25}$$

where Nu is the Nusselt number based on the particle diameter, and the particle Reynolds number depends on the instantaneous magnitude of the slip velocity S between the gas and the particle.

Because Nu depends explicitly on the instantaneous slip, it is conveniently averaged using a velocity distribution function f(S). This function is normalized so that its integral over the entire velocity space equals the number density of particles n at the point under consideration

$$\iiint f(\mathbf{S})\, d\mathbf{S} \equiv n = (1-\varepsilon)/(\pi/6)d^3. \tag{26}$$

Using this distribution, the average $\langle\psi\rangle$ of any function of the slip $\psi(\mathbf{S})$ is

$$\langle\psi\rangle = \frac{1}{n}\iiint \psi(\mathbf{S})\, f(\mathbf{S})\, d\mathbf{S}. \tag{27}$$

In the present treatment we assume that this average is equivalent to ensemble or time averages employed for the gas and particle phases. To evaluate the source term, we multiply the rate of energy transferred from each particle to the gas by the total number of particles per unit volume. Upon averaging the result, we obtain

$$H = (1-\varepsilon)\frac{6k_g}{d^2}(T_p - T_g)\langle Nu\rangle. \tag{28}$$

To average the fractional powers of Re arising in (25), we write the magnitude of the instantaneous slip Reynolds number in the form

$$Re = \left(\frac{\rho d\sqrt{(s^2+2k+3\Theta)}}{\mu}\right)[1+\xi(\mathbf{s}';k,\Theta,\mathbf{s})]^{1/2} \tag{29}$$

where $\mathbf{s} = (\mathbf{u}-\mathbf{v})$ is the average slip along the pipe axis, $\mathbf{s}'$ the fluctuating slip, and

$$\xi(\mathbf{s}';k,\Theta,\mathbf{s}) \equiv \frac{(2\mathbf{s}\cdot\mathbf{s}'+s'^2-2k-3\Theta)}{(s^2+2k+3\Theta)}. \tag{30}$$

Upon integrating the binomial expansion of $(1+\xi)^{a/2}$ through the velocity distribution function, we find

$$\langle Re^a\rangle = \left(\frac{\rho d\sqrt{(s^2+2k+3\Theta)}}{\mu}\right)^a \times\left[1+\frac{a}{2}\langle\xi\rangle+\frac{a}{4}\left(\frac{a}{2}-1\right)\langle\xi^2\rangle+o(\langle\xi^3\rangle)\right]$$

so, upon ignoring terms of order higher than the first

$$\langle Re^a\rangle \approx \left(\frac{\rho d\sqrt{(s^2+2k+3\Theta)}}{\mu}\right)^a\left[1-a\frac{\langle u_i'v_i'\rangle}{s^2+2k+3\Theta}\right]. \tag{31}$$

Finally, with $a = 1/2$ or $2/3$, respectively, we have

$$\langle Nu\rangle \approx \left[2+0.4Pr^{0.4}\left(\frac{\rho d\sqrt{(s^2+2k+3\Theta)}}{\mu}\right)^{1/2} \times\left(1-\frac{1}{2}\frac{\langle u_i'v_i'\rangle}{s^2+2k+3\Theta}\right)+0.06Pr^{0.4} \times\left(\frac{\rho d\sqrt{(s^2+2k+3\Theta)}}{\mu}\right)^{2/3} \times\left(1-\frac{2}{3}\frac{\langle u_i'v_i'\rangle}{s^2+2k+3\Theta}\right)\right] \tag{32}$$

where an estimate of $\langle u_i'v_i'\rangle$ is provided by (11). Because the contribution of this term is relatively small, the rate of heat transfer between the particles and the gas is influenced primarily by the mean slip.

3.4. *Fully-developed heat transfer*

To discuss the mechanisms of heat transfer in this regime, we focus on two separate examples. In the first, a constant heat flux q_w is imposed at the wall. In the second, a constant temperature T_w is maintained there. In either case, the thermal energy flux carried by the particles vanishes at the wall. Then, because the self-diffusive conductivity of the particles is finite, the thermal temperature of the particle phase satisfies the homogeneous boundary condition at the wall

$$\frac{\partial T_p}{\partial r} = 0. \tag{33a}$$

In the gas phase, because the turbulent diffusivity vanishes at the wall

$$\varepsilon k_g\frac{\partial T_g}{\partial r} = q_w \tag{33b}$$

where q_w remains an unknown obtained by iteration when a constant T_w is imposed. At the centerline, radial symmetry requires $\partial T_p/\partial r = \partial T_g/\partial r = 0$.

In order to define what is meant by fully developed thermal flows in these two cases, we first introduce the total heat capacity rate M and the average bulk temperature T^b

$$M \equiv \int_0^R [\rho cu\varepsilon+\rho_p c_p v(1-\varepsilon)]2\pi r\,\mathrm{d}r = cQ\left(1+m\frac{c_p}{c}\right) \tag{34}$$

and

$$T^b \equiv \frac{1}{M}\int_0^R [\rho cu\varepsilon T_g+\rho_p c_p v(1-\varepsilon)T_p]2\pi r\,\mathrm{d}r \tag{35}$$

where Q is the average mass flow rate of gas, and m the average loading. In steady turbulent flow, a cross-sectional heat balance shows that the rate of change of the instantaneous bulk temperature along the pipe is related to the heat flux across the wall, so on average

$$\frac{\mathrm{d}T^b}{\mathrm{d}z} \approx \frac{2\pi Rq_w}{M}. \tag{36}$$

This balance is nearly exact; because instantaneous velocity and temperature fluctuations are much smaller than their time-averaged values, we ignore in this analysis temporal moments of order higher than the first in the expressions for T^b and M.

We define the average heat transfer coefficient h at the wall as

$$h(T_w - T^b) = q_w. \tag{37}$$

Fully developed thermal flows are defined as flows in which the temperature profiles are similar, i.e. the average normalized temperatures

$$\theta_g \equiv (T_g - T_w)/(T^b - T_w) \tag{38a}$$

$$\theta_p \equiv (T_p - T_w)/(T^b - T_w) \tag{38b}$$

are independent of z. Because from (33b), (37) and (38a)

$$h = -\varepsilon k_g\left(\frac{\partial\theta_g}{\partial r}\right)_{r=R}$$

it follows that h is also independent of z.

When the heat flux q_w is constant at the wall

$$\frac{\partial T_g}{\partial z} = \frac{\partial T_p}{\partial z} = \frac{dT^b}{dz} \approx \frac{2\pi R q_w}{M};$$

so

$$\frac{\partial^2 T_g}{\partial z^2} = \frac{\partial^2 T_p}{\partial z^2} = 0$$

and the balance laws (20) and (23) become ordinary differential equations (ODEs) in r. Thus for the gas

$$\varepsilon\rho c u \frac{dT_w}{dz} = \frac{1}{r}\frac{d}{dr}(rQ_r) + H; \tag{39}$$

and for the particles:

$$(1-\varepsilon)\rho_p c_p v \frac{dT_w}{dz} = \frac{1}{r}\frac{d}{dr}(rQ_r^p) - H \tag{40}$$

where $dT_w/dz = dT^b/dz$ is a constant and Q_r and Q_r^p are given by (21a) and (24a).

For constant wall temperature, through differentiation of (36) and (38) the balance laws (20) and (23) may be rearranged as two coupled ODEs. Thus for the gas:

$$\varepsilon\rho c u \theta_g \frac{2\pi R}{M} q_w = \frac{1}{r}\frac{d}{dr}(rQ_r) + \frac{\partial}{\partial z}(Q_z) + H \tag{41a}$$

with

$$\frac{\partial}{\partial z}(Q_z) = -(k_g + \rho c \varepsilon_H)\varepsilon\theta_g\left(\frac{2\pi R}{M}\right)^2 \frac{q_w^2}{T_w - T^b} \tag{41b}$$

and Q_r given by (21a); and for the particles:

$$(1-\varepsilon)\rho_p c_p v \theta_p \frac{2\pi R}{M} q_w = \frac{1}{r}\frac{d}{dr}(rQ_r^p) + \frac{\partial}{\partial z}(Q_z^p) - H \tag{42a}$$

with

$$\frac{\partial}{\partial z}(Q_z^p) = -\frac{\sqrt{\pi}}{16}\rho_p c_p d\sqrt{\Theta}\theta_p\left(\frac{2\pi R}{M}\right)^2 \frac{q_w^2}{T_w - T^b}\frac{1}{1+\lambda/R} \tag{42b}$$

and Q_r^p given by (24a). Because q_w is unknown a priori, with constant wall temperature the solution of (41) and (42) must be found by iteration.

In the hydrodynamic analysis the governing equations for the flow give rise to eight dimensionless numbers. The loading m and the Reynolds number Re_D based on the superficial gas velocity and the pipe diameter D define the flow conditions. Combinations of the particle and gas properties produce the Archimedes number $Ar = \rho_p \rho g d^3/\mu^2$ and the density ratio ρ_p/ρ. The size of the particles relative to that of the pipe is D/d. Finally, the coefficients e, e_w and μ_f are dimensionless collision properties. In addition, the Prandtl number of the gas and the ratio c_p/c arise from the thermal equations (20) and (23). Because of the boundary condition (33b), it is convenient to make the temperatures of both phases dimensionless through the product $(q_w R/k_g)$. In this case, the slope of the dimensionless gas temperature profile vs relative radius r/R is always $(1/\varepsilon) \sim 1$ at the wall. Finally, because heat transfer at the wall occurs through the gas phase alone, the Nusselt number $Nu_D \equiv hD/k_g$ is a natural measure of h.

4. RESULTS AND DISCUSSION

The balance equations for fully developed heat transfer are first written as a set of four coupled, linear, first-order ODEs. These equations are integrated from the centerline of the pipe to the wall using a fourth-order Runge–Kutta algorithm, combined with an iterative Newton–Raphson scheme to satisfy the boundary conditions at the wall.

Because the closure model (14) and (15) is only valid in the turbulent core, in the thermal analysis we use values of the flow parameters that are extrapolated through the relatively thin buffer and viscous sublayers. In particular, because the particle velocity and granular temperature have finite values at the wall, these are extrapolated assuming a constant gradient through that region. This assumption is rather inconsequential, because for small friction, the predicted heat transfer rates are relatively insensitive to the coefficients of particle friction and restitution and to large variations in the self-diffusive conduction fluxes (24a) and (24b) of the particle phase. In addition, for the flows under consideration, variations in the form of the turbulent kinetic energy profile near the wall do not affect the trends predicted by the thermal analysis. As a result, we simply assume that k varies linearly with the radius between its predicted value at $y^+ = 30$ and zero at the wall. In the future, a refined thermal analysis could invoke near-wall turbulence models (see for example Patel et al. [21]). However, to be meaningful, such a refinement should also treat the effect of massive particles on the near-wall turbulence.

To highlight the parameters that govern the heat transfer in this regime, we focus on the case where a constant heat flux is imposed at the wall, and we illustrate the discussion by integrating equations (39) and (40) with the help of simplifying hypotheses. However, note that the results presented in the figures are obtained through the numerical integration of the complete ODEs. We begin with a discussion of the effects of introducing particles in the flow at increasingly higher loadings. Then we discuss the effect of the relative size of the pipe and the particles. Figure 3 shows typical temperature profiles across the pipe. In general, the particles affect heat transfer in two opposite ways.

First, they generally reduce the turbulent kinetic energy of the gas [8], unless they are so very massive that turbulence is augmented, perhaps by their own wakes or by deflections of the mean flow streamlines induced by the particles [16]. Because at relatively small loading the source term is dominated by the turbulent conduction term in the gas energy equation

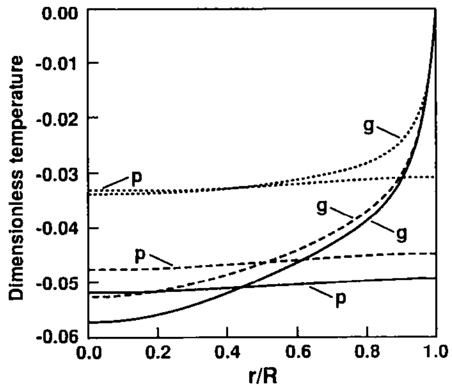

FIG. 3. Typical average gas (g) and particle (p) temperature profiles with constant q_w for the flow conditions $Re_D = 15\,000$, $\rho_p/\rho = 2100$, $Ar = 814$, and $R/d = 90$. Temperature is made dimensionless through the product $(q_w R/k_g)$. The thermal parameters are typical of suspensions of glass particles in air with $c_p/c = 0.8$ and $Pr = 0.7$. The solid lines are predictions of the complete analysis for $m = 0.4$. In this case, $h/h_0 = 1.01$, where h_0 is the heat transfer coefficient for the same flow of clear gas. The dashed and dotted lines correspond to $m = 0.4$ and 3.0, respectively. These lines are hypothetical predictions from a numerical model that artificially ignores the turbulence reduction associated with the introduction of particles.

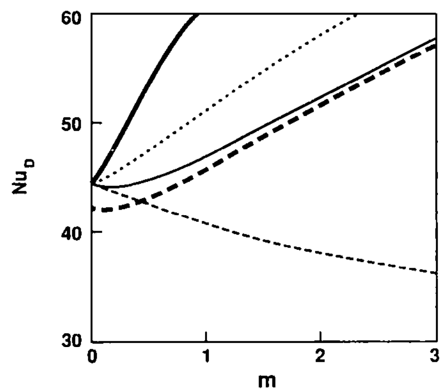

FIG. 4. Predictions of the analysis for Nusselt number Nu_D vs loading m for the conditions of Fig. 3. The solid and heavy dashed lines are predictions of the thermal analysis with constant q_w and constant T_w, respectively. The dotted and thin dashed lines are hypothetical predictions of numerical models that ignore the gas turbulence reduction and the particle self-diffusive conduction, respectively. The heavy line corresponds to small particles with $D/d = 250$.

(39), the gas heat flux Q_r remains nearly unchanged for small m, so lower turbulent transport coefficients result in higher gradients of gas temperature in the pipe. Consequently, as Fig. 3 indicates, the gas temperature predicted by the complete analysis (solid line) is lower than if turbulence were not reduced (dashed line). In addition, because of its coupling through the source term, the particle temperature also decreases with decreasing turbulence intensity.

Second, particles promote the radial transfer of thermal energy through particle self-diffusive conduction. Consequently, through the source term H, they increase the temperatures of both phases with increasing loading. To illustrate this effect alone, Fig. 3 shows hypothetical temperature profiles for two loadings (dashed and dotted lines), where the turbulence intensity is artificially kept identical to that of clear gas. In the absence of turbulence reduction, as the dotted line in Fig. 4 indicates, the heat transfer coefficient would rapidly increase with m.

Therefore, particles have competing effects on the temperature profiles. On the one hand, they lower these profiles through turbulence reduction in the gas, and on the other, they increase them through particle self-diffusive conduction and the source term. The resulting changes in the difference $(T_w - T^b)$ between the wall and bulk temperatures affect the heat transfer coefficient in (37). For loadings small enough that $(mc_p/c) \ll 1$, the total heat capacity rate M in (34) is approximately equal to cQ. In this case, through (35), the difference between the wall and bulk temperatures is approximately

$$(T_w - T^b) \approx (T_w - T_g^b) + m\frac{c_p}{c}(T_w - T_p^b) \quad (43)$$

where T_g^b and T_p^b represent the cross-sectional average temperatures for each of the two phases:

$$T_g^b \equiv \frac{1}{Q}\int_0^R \rho u \varepsilon T_g 2\pi r \, dr \quad (44a)$$

$$T_p^b \equiv \frac{1}{mQ}\int_0^R \rho_p v(1-\varepsilon) T_p 2\pi r \, dr. \quad (44b)$$

In most instances at small loading, the effects of turbulence reduction in the gas dominate or balance the effects of particle self-diffusive conduction, so $(T_w - T_g^b)$ and $(T_w - T_p^b)$ increase with m, or at least become independent of it. Therefore, through (43), $(T_w - T^b)$ increases with m, so the heat transfer coefficient generally decreases with m at small loading.

This trend would persist at higher values of m in the absence of a particle self-diffusive conduction flux. In this hypothetical—and rather unrealistic—case where self-diffusive conduction would vanish, equation (40) yields the following algebraic expression for the dimensionless temperature difference between the two phases:

$$\frac{T_g - T_p}{q_w R/k_g} = \frac{\rho_p}{\rho}\frac{c_p}{c}\frac{v}{\overline{\varepsilon u}}\left(\frac{d}{R}\right)^2 \frac{1}{3\langle Nu\rangle(1 + c_p m/c)} \quad (45)$$

where the overbar denotes the cross-sectional average and $\langle Nu\rangle$ is given by (32). For typical conditions, this difference is small at any radial position. Therefore, if we assume for simplicity that $[\rho c\varepsilon u + \rho_p c_p(1-\varepsilon)v]$ is constant across the pipe, the sum of equations (39) and (40) would integrate to

$$(T_w - T_g) \approx \frac{q_w}{\rho c}\int_{r/R}^{1} \frac{\zeta \, d\zeta}{\varepsilon(\varepsilon_H + \alpha)} \quad (46)$$

where α is the molecular heat diffusivity of the gas and, because in this case $(T_w - T_g^b) \approx (T_w - T_p^b) \approx$

$(T_w - T^b)$, any reduction in ε_H would increase $(T_w - T^b)$ and reduce h. Therefore, in the absence of particle self-diffusive conduction, the heat transfer coefficient would decrease as long as turbulence intensity decreased with loading (thin dashed line in Fig. 4). Because the opposite behavior is generally observed at relatively high loading, this discussion suggests that particle self-diffusive conduction is an essential mechanism of heat exchange in these flows.

In fact, with particle self-diffusive conduction, the effects of reduction in the gas turbulence intensity become less significant as loading increases. The direct integration of either equation (39) or (40) yields

$$Nu_D = 6\left(\frac{R}{d}\right)^2 \langle Nu \rangle \frac{\overline{(1-\varepsilon)(T_g - T_p)}}{T_w - T^b}\left(1 + \frac{c}{c_p m}\right) \tag{47}$$

where the overbar denotes the cross-sectional average. Assuming uniform profiles of ε, u and v, $(1-\bar{\varepsilon}) \ll 1$ and $mc_p/c \gg 1$, this equation simplifies to

$$Nu_D \approx m\langle Nu \rangle 6\left(\frac{R}{d}\right)^2 \frac{\rho}{\rho_p}\frac{\bar{u}}{\bar{v}}\left(\frac{T_g^b - T_p^b}{T_w - T^b}\right). \tag{48}$$

Our experience in solving equations (39) and (40) numerically is that the temperature ratio in (48) is a weak function of m for values of m large enough that relative changes in turbulent intensity are insignificant. As equation (48) shows, in this limit the heat transfer coefficient grows linearly with m, and the growth is more pronounced for smaller particles or larger pipes (Fig. 4, heavy line). By inspection, it is clear that the source term is responsible for this through the factors $(R/d)^2$ and $\langle Nu \rangle$ in (48). Therefore, in the absence of a source term, the transport of a conserved scalar would not be enhanced by the introduction of particles with high loading, despite the presence of significant particle self-diffusion. Ebert *et al.* [22] made a similar observation in the different regime of circulating fluidization. There, they showed that, although heat transfer is greatly enhanced by particles, the rate of mass transfer from a solid block of naphthalene mounted flush with the bed wall is no greater than in the absence of particles.

In the regime under consideration, the relative size D/d of the pipe and the particles affects the wall heat transfer primarily through the two competing mechanisms of gas turbulence reduction and particle self-diffusive conduction. First, through the drag term in (13) and the working of the mean gas shear in (16), the production of gas turbulence is reduced in the interior with increasing D/d. Consequently, in the absence of particle self-diffusive conduction, the wall heat transfer would decrease with greater D/d. However, because the source term grows approximately with $(D/d)^2$, larger values of this ratio also produce a greater exchange of heat between the particles and the gas and, consequently, the presence of particle self-diffusive conduction results in a flatter gas temperature profile and greater wall heat transfer. This effect, which dominates turbulence reduction at relatively high loadings, is captured by equation (48). At these loadings, it explains why Boothroyd and Haque [3] observed that heat transfer coefficients are increased by the solids far more in larger pipes than smaller ones.

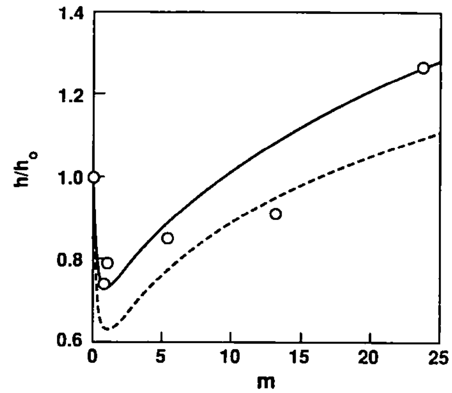

FIG. 5. Predictions of the analysis compared with the data of Jepson *et al.* [1] for sand particles of 422 μm $\leqslant d \leqslant$ 599 μm suspended by air in a pipe of 38 mm in diameter. Here, $Re_D = 45\,000$, $\rho_p/\rho = 2100$, $6300 \leqslant Ar \leqslant 18\,000$, $c_p/c = 0.8$ and $Pr = 0.7$. The solid and dashed lines are predictions for 422 and 599 μm, respectively. For these conditions, $\Theta^{1/2}$ is approximately 1% of the gas velocity at the centerline.

In general, the present thermal analysis provides explanations for all trends summarized by Boothroyd and Haque [3] for the regime under consideration. For example, these authors reported a sharper drop of heat transfer coefficient at low loading for higher values of Re_D. The relatively greater reduction of turbulence associated with increasing Re_D provides the mechanism responsible for this. Further discussions of the flow parameters and suspension properties affecting wall heat transfer in this regime are provided by Mohd. Yusof [23].

Figure 5 compares predictions of this analysis with the experimental data of Jepson *et al.* [1]. In those experiments, particle size ranges from 422 μm to 599 μm. For these particles, the Biot number $Bi \equiv hd/k_p$ and Rc are of order 10^{-2} and 10^9, respectively, and the ratio of particle hydrodynamic relaxation time to a typical roll-over time of turbulent eddies is approximately 20. Because heat transfer is a strong function of particle diameter through the source term H, we bracket the data with predictions corresponding to the limits of the experimental size distribution. As Fig. 5 indicates, the predictions are satisfactory, and they exhibit the minimum heat transfer coefficient observed at low values of loading. The calculations also demonstrate the importance of measuring the correct particle size in the experiments.

5. CONCLUSIONS

In this paper, we have analyzed the fluid dynamics and heat transfer of fully developed suspensions of massive particles in a vertical pipe. In this regime, we have shown that, because they introduce shear stress in the particle phase, collisions cannot be ignored even in relatively dilute suspensions. In addition, although they are too rapid to permit the direct exchange of heat between particles and the wall, collisions drive the mechanism of particle self-diffusive conduction that flattens the particle temperature profile and thus maintains a significant temperature difference between the gas and the particle phases. Therefore as loading increases, the two phases exchange greater amounts of heat that produce growing heat transfer coefficients through flatter gas temperature profiles. In contrast, at low loading, the heat exchange between the two phases is too small to enhance the wall heat transfer despite the presence of particle self-diffusive conduction. In that case, because their introduction generally dissipates turbulence, massive particles reduce the heat transfer to the wall.

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